TRINITY COLLEGE

Mathematics Methods U 3,4
Test 1 2022

Section 1 Calculator Free

Differentiation, Applications of Differentiation and Antidifferentiation

STUDENT'S NAME Solutions - WILSON.

DATE: Wednesday 2nd March **TIME:** 25 minutes **MARKS:** 28

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser, formula sheet

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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1. (5 marks)

Differentiate the following expressions with respect to x with use of the rule indicated. Do not simplify your answer.

(a) $\left(3x-4x^2\right)\left(2x^3+3x-1\right)$ (product rule) [2]

$$\frac{dy}{dx} = (3-8x)(2x^3+3x-1) + (3x-4x^2)(6x^2+3)$$

✓ ✓

(b) $\dfrac{\left(5-x^2\right)^4}{\sqrt{x}+5}$ (quotient rule) [3]

$$\frac{d}{dx} = \frac{4(5-x^2)^3.(-2x).(x^{1/2}+5) - (\frac{1}{2}x^{-1/2})(5-x^2)^4}{(x^{1/2}+5)^2}$$

✓ correctly uses chain rule

✓ denominator of quotient rule

✓ numerator of quotient rule.

2. (11 marks)

(a) Determine the following:

(i) $\int \frac{3x^2+\sqrt{x}}{x}\,dx$ [2]

$\int 3x + x^{-1/2}\,dx$

$= \frac{3}{2}x^2 + 2x^{1/2} + c$

✓ simplifies eq.

✓ antidifferentiates

(ii) $\int x(2x-1)^2\,dx$ [3]

$\int x(4x^2-4x+1)\,dx$

$\int 4x^3 - 4x^2 + x\,dx.$

$= x^4 - \frac{4}{3}x^3 + \frac{x^2}{2} + c$

✓ expands perfect square

✓ expands function completely

✓ antidifferentiates

(iii) $\int \frac{2}{5(3x+8)^5}\,dx$ [3]

$\frac{2}{5}\cdot\frac{1}{3}\int 3(3x+8)^{-5}\,dx$

$\frac{2(3x+8)^{-4}}{15(-4)} + c$

$= \frac{-1}{30(3x+8)^4} + c$

✓ takes out coefficient $\frac{2}{5}$

✓ divides by 3.

✓ correct antiderivative.

(b) Given that $f'(x)=(2-4x)^4$ and $f(1)=-2$ determine the equation of $f(x)$. [3]

$f(x) = \frac{-1}{4}\int -4(2-4x)^4\,dx.$

$= \frac{-(2-4x)^5}{20} + c$

$-2 = \frac{-(2-4(1))^5}{20} + c.$

$c = -\frac{18}{5}$

$f(x) = \frac{-(2-4x)^5}{20} - \frac{18}{5}$

✓ antidifferentiates

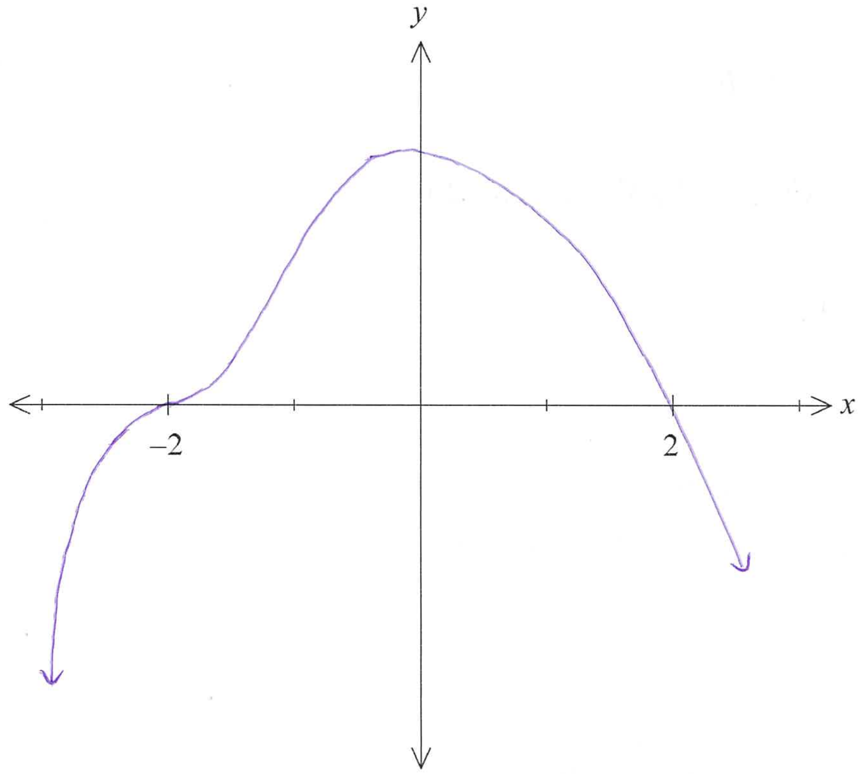
✓ subs in coordinate

✓ solves for c & states equation of f(x)

3. (4 marks)

Sketch a function $y = f(x)$ on the axes provided with all the following features.

- $f(2) = f(-2) = 0$
- $f'(-2) = f'(0) = 0$
- $f''(-2) = 0$
- $f'(x) > 0$ for $-2 < x < 0$
- $f'(x) < 0$ for $x > 0$



✓ roots at $x = -2$ & 2

✓ stationary points at $x = -2$ & 0

✓ P.O.I at $x = -2$

✓ Maximum T.P at $x = 0$

4. (8 marks)

(a) The curve $y = x^3 - ax^2 + bx + c$ has a y-intercept of 5 and the gradient at that point is 6. If the curve passes through the point $(2,13)$, determine the values of a, b, and c. [3]

$c = 5$

✓ identifies $c=5$

$\frac{dy}{dx} = 3x^2 - 2ax + b$

✓ differentiates, & subs in $\frac{dy}{dx} = 6$ at $x=0$ and solves $b=6$

$6 = 3(0)^2 - 2a(0) + b$

$\therefore b = 6$

$13 = (2)^3 - a(2)^2 + 6(2) + 5$

✓ subs in b, c and coordinate into function and solves $a=3$.

$4a = 12$

$a = 3$

(b) A tangent to the curve $y = 8\sqrt{x} + \frac{x}{2} - 4$ is drawn at point T. If the tangent is parallel to the line $-3x + 2y = 7$, determine the equation of the tangent to the curve at point T. [5]

$m = \frac{3}{2}$

$y = 8x^{1/2} + \frac{1}{2}x - 4$

$\frac{dy}{dx} = 4x^{-1/2} + \frac{1}{2}$

$\frac{3}{2} = \frac{4}{\sqrt{x}} + \frac{1}{2}$

$\sqrt{x} = 4$

$\therefore x = 16$

$y = 8\sqrt{16} + \frac{16}{2} - 4$

$= 36$

$y = \frac{3}{2}x + c$

$36 = \frac{3}{2}(16) + c$

$c = 12$

$\therefore y = \frac{3}{2}x + 12$

✓ determines gradient of $\frac{3}{2}$

✓ differentiates function

✓ solves for x and y

✓ writes equation $y = \frac{3}{2}x + c$ with sub $(16, 36)$

✓ solves c and states equation.

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Mathematics Methods Unit 3,4
Test 1 2022

Section 2 Calculator Assumed
Differentiation, Applications of Differentiation and Antidifferentiation

STUDENT'S NAME ______________________________

DATE: Wednesday 2nd March **TIME:** 25 minutes **MARKS:** 24

INSTRUCTIONS:

Standard Items: Pens, pencils, drawing templates, eraser, formula sheet

Special Items: Three calculators, notes on one side of a single A4 page (these notes to be handed in with this assessment)

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

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5. (5 marks)

A probe moving through the solar system uses a solar panel to charge its batteries. The number of millivolts the panel generates depends on the distance in millions of kilometres, x, from the sun. The relationship can be described by the equation:

$$f(x) = \frac{4000}{x} + \frac{8000}{x^2} + 400$$

(a) Determine the equation for the rate of change in voltage generated per million kilometres, when the probe is x million kilometres from the sun. [1]

$f'(x) = \dfrac{-4000x + 16000}{x^3}$

(b) Determine the change in voltage generated per million kilometres, when the probe is 6×10^8 kilometres from the sun to 4 decimal places. [2]

$x = 600$ $f'(600) = -0.0112$ volts/million km

✓ $x = 600$

✓ $f'(600)$

(c) Determine the change in voltage generated per million kilometres, when the probe moves from 6×10^8 kilometres from the sun to 10^9 kilometres from the sun to 4 decimal places. [2]

$\dfrac{f(1000) - f(600)}{400}$

$= 0.0067$ volts/million km.

✓ $x = 1000$ & 600

✓ calculates avg. rate of change.

6. (7 marks)

The cost in dollars of producing x items of a product is given by $C(x) = 3000 + 5x$.

The revenue per item sold is given by the expression $\$\,40 - 0.02x$.

(a) Give the equation of the profit, $P(x)$, and simplify. [2]

$P(x) = x(40 - 0.02x) - (3000 + 5x)$

$= -0.02x^2 + 35x - 3000$

✓ establishes equation

✓ simplified

(b) Determine how many items are needed to make a maximum profit and the maximum profit. [2]

$x = 875$ items

$P(x) = \$\,12312.50$

✓ determines x

✓ determines $P(x)$.

(c) Determine the marginal profit of the 250th item sold. [3]

$P'(x) = -0.04x + 35$

$\Delta P = -0.04(250) + 35$

$= \$25$

✓ differentiates $P(x)$

✓ subs in 250

✓ evaluates $P'(250)$

7. (7 marks)

A company wishes to design cylindrical metal containers with a volume of 16 cubic metres. The top and the bottom will be made of a sturdy material which costs \$2 per square metre, while the material for the side's costs \$1 per square metre.

(a) Determine the equation for the cost of the container, C, in terms of the radius, r. [3]

$16 = \pi r^2 h$

$\therefore h = \frac{16}{\pi r^2}$

$SA = 2\pi r^2 + 2\pi r\left(\frac{16}{\pi r^2}\right)$

$= 2\pi r^2 + \frac{32}{r}$

$Cost = 4\pi r^2 + \frac{32}{r}$

✓ establishes h

✓ establishes SA.

✓ establishes Cost.

(b) Showing use of calculus techniques, determine the radius, height, and cost of the cheapest container possible. Give the lengths to 4 decimal places and cost to the nearest cent. [4]

$\frac{dC}{dr} = \frac{8\pi r^3 - 32}{r^2}$

$0 = \frac{8\pi r^3 - 32}{r^2}$

$r = 1.0839$ m.

$h = \frac{16}{\pi(1.0839)^2}$

$= 4.3350$ m.

$C = 4\pi(1.0839)^2 + \frac{32}{1.0839}$

$= \$44.29$.

✓ differentiates C

✓ equates $\frac{dC}{dr}$ to 0 and solves for r.

✓ evaluates h

✓ evaluates C.

8. (5 marks)

The ratio of the radius, r, to the height, h, is $5:3$ for a specific cone. The cone is to be filled with water to a depth of h cm.

(a) Show that the volume of the cone is given by $V = \dfrac{25\pi h^3}{27}$. [2]

$r = \frac{5}{3}h$

$V = \frac{1}{3}\pi\left(\frac{5h}{3}\right)^2 h$

$= \dfrac{25\pi h^3}{27}$

✓ establishes $r = \frac{5}{3}h$

✓ subs in $r = \frac{5}{3}h$ into volume of cone.

(b) Determine the approximate increase in the volume of liquid in the cone if the depth increases from 5 cm to 5.02 cm to 2 decimal places. [3]

$\dfrac{dV}{dh} = \dfrac{25\pi h^2}{9}$.

$\Delta V \approx \dfrac{25\pi(5)^2}{9} \times 0.02$.

$\approx 4.36 \text{ cm}^3$

✓ differentiates ✓

✓ substitutes in $h = 5$ and $\Delta h = 0.02$

✓ evaluates $\Delta V \approx$